Incorporation of Diffraction Effects in Simulations of Ultrasonic Systems using PSpice models

Jonny Johansson  
Luleå University of Technology  
971 87 Luleå, Sweden

Per-Erik Martinsson  
Luleå University of Technology  
971 87 Luleå, Sweden

Abstract – The use of PSpice models for piezoelectric devices and ultrasonic transmission media is of major importance when designing electronics for ultrasonic systems. Today, these models include viscoelastic loss but disregard loss due to diffraction, i.e. beam spreading. This paper presents a method for the inclusion of diffraction loss in PSpice simulations of ultrasonic systems. The conductive loss in the transmission line that models the propagation media of the ultrasound pulse is used to model the loss due to diffraction. Parameter variations for the piezoelectric device can affect the result greatly. Thus, a sensitivity analysis for the simulation model is also presented.

Measurements and simulations have been performed using a pulse echo system in water. Maximum distance to the reflector was 200 mm. The piezoelectric devices used are PZ-27 crystals with diameters 6 mm and 12mm, with a center frequency of 4 MHz. Results show that the simulated amplitude of the returned echo differs less than 10% from measured values.

I. INTRODUCTION

In the design of ultrasonic systems the use of simulation tools intended for electronic circuits (e.g. SPICE, PSpice) is well established. Equivalent circuits for piezoelectric elements using only passive components were initially developed in [1 – 3]. Leach [4] used controlled sources to avoid negative capacitance and frequency dependant transformers. Püttmer [5] included losses in this model to model low-Q thickness mode transducers. Also, losses were introduced in the transmission media to give the attenuation of the propagating wave. Van Deventer [6] further investigated material properties governing viscoelastic loss and wave speed. However, none of the works hitherto presented has concerned simulations giving absolute amplitudes of received signals. When designing electronics for an ultrasonic system, absolute amplitudes are of importance as they set required dynamics in the electronic system. One important factor when aiming for correct amplitude is the loss introduced in a system due to diffraction, i.e. beam spreading. The paper at hand presents a method of including diffraction loss in a pulse echo simulation in order to achieve absolute amplitude correctness. Material parameters used have a large influence on received echo amplitudes. Therefore, a sensitivity analysis for the simulation model is presented.

II. METHOD

Modeling the diffraction effect

When simulating an ultrasonic system using PSpice, electrical transmission lines are used as the propagation medium [5, 6]. Under the assumption of low-loss conditions ($R << \omega L, G << \omega C$), the attenuation constant $\alpha$ for an electrical transmission line can be written [6]:

$$
\alpha = \frac{1}{2} \sqrt{LC} \left( \frac{R}{L} \right) + \frac{1}{2} \sqrt{LC} \left( \frac{G}{C} \right)
$$

(1)

Here $R$ is resistive loss, $G$ is conductive loss, $C$ is capacitance and $L$ is inductance per unit length. This can be rewritten

$$
\alpha = \frac{1}{2 \, Z_0} \left( \frac{R}{Z_0} \right) + \frac{1}{2} \frac{GZ_0}{C}
$$

(2)

where $Z_0 = \sqrt{LC}$. The propagating sound wave is modelled as a forward travelling voltage wave in the transmission line. The amplitude of this voltage wave can be expressed as [7]:

...
where $V_0$ is the voltage amplitude at $x=0$. Using (2) we get

$$|V(x)| = |V_0|e^{-(Rx/Z_0)x} e^{-(GZ_0/2)x}. \quad (4)$$

Previous works on simulation of ultrasound propagation using electric transmission lines have used $R$ for modelling viscous losses while setting $G$ to 0 [5, 6]. Here, by using $G \neq 0$ we can add an attenuation term non-dependent of the attenuation caused by $R$. This can be used to model the attenuation due to diffraction as

$$A_{diff} = \frac{|V(x)|}{|V_0|e^{-(Rx/Z_0)x}} = e^{-(GZ_0/2)x}. \quad (5)$$

Solving for $G$ gives

$$G = \frac{2}{Z_0 x} \ln(A_{diff}), \quad (6)$$

which can be used as a parameter in PSpice once the required diffraction loss is known.

Given two identical transducers with radius $a$ spaced a distance $x$ apart, the diffraction loss when transmitting from one to another can be expressed as [8, 9]

$$A_{loss} = \left[ 2 \int_0^\infty J_J^2(y) e^{inS/4\pi} dy \right]. \quad (7)$$

Here

$$S = x\lambda/a^2 \quad (8)$$

is the Seki parameter, with $\lambda$ being the wavelength of the transmitted ultrasound pulse. The integrand

$$y = k_j a \quad (9)$$
in which $k_j$ is the radial propagation constant. For a pulse echo system with a perfect plane reflector, $x$ in (8) is replaced with $2x$, where $x$ is the distance to the reflector [8]. $A_{loss}$ as given by (7) is plotted for $1 \leq S \leq 30$ in figure 2.

Figure 1: Schematic of the simulation
As equation (7) must be solved numerically, the use of it as a parameter in PSpice simulations is not straightforward. From figure 2 it is seen that the behaviour of (7) can be approximated with two straight lines in a log-log scale. Thus, an analogy with electrical filters was used to create an approximation of the diffraction loss. Two “filter” sections were used, one to create a straight line for \( S \leq 4 \),

\[
A_1 = \frac{1}{1.05 \cdot (S/0.1)^{0.07}},
\]

and one to increase the slope for \( S \geq 4 \):

\[
A_2 = \left[ (1 + S/4)^{3.186} \right].
\]

The total attenuation is given by

\[
A_{\text{tot}} = A_1 \cdot A_2.
\]

The exponents for the denominators in (10) and (11) are chosen to give \( A_{\text{tot}} \) a slope of 1/S in the far field for \( S > 4 \), following the Fraunhofer approximation [8]. The order of the “filter section” in (11) is chosen to match the roll-off to the curve given by (7). \( A_{\text{tot}} \) is plotted for \( 1 \leq S \leq 30 \) in figure 2. It is seen here that the small variations in attenuation for \( S \leq 2 \) are excluded in the approximation. If these were to be in-cluded, the model could easily be extended using more “filter sections” containing higher order terms of \( S \). In this paper, they have been omitted for clarity. Using (6) and (12) and setting

\[
A_{\text{diff}} = A_{\text{tot}}
\]

now gives a complete set of input parameters for the modelling of diffraction loss.

The method described above is applicable only if the diffraction occurs mainly in one medium, i.e. if other parts of the propagation path are thin enough that diffraction in these can be considered negligible.

**Simulation setup**

The schematic used for simulation is shown in figure 1. Simulations were performed using the Cadence Spectre simulation engine. Spectre has the possibility to run netlists created for PSpice, enabling the use of models previously developed for PSpice. The model used for the simulation of the piezo-ceramic disc was initially presented by Leach [4], further developed by Pütterm [5] and van Deventer [6]. Also the media through whom the ultrasonic

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>( a ) (m)</td>
<td>0.006 / 0.012</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho ) (kg/m(^3))</td>
<td>7700</td>
</tr>
<tr>
<td>Thickness</td>
<td>( len ) (m)</td>
<td>500e-6</td>
</tr>
<tr>
<td>Permittivity</td>
<td>( \varepsilon^\prime ) (As/Vm)</td>
<td>8.09e-9</td>
</tr>
<tr>
<td>Piezoelectric constant</td>
<td>( \varepsilon^\prime\prime ) (C/m(^2))</td>
<td>16</td>
</tr>
<tr>
<td>Q factor</td>
<td>( Q )</td>
<td>74</td>
</tr>
<tr>
<td>Conductivity</td>
<td>( G ) (1/( \Omega ))</td>
<td>0</td>
</tr>
<tr>
<td>Elastic constant</td>
<td>( C ) (Pa)</td>
<td>1.13\times10^{11}</td>
</tr>
</tbody>
</table>

Table 1: Simulation data for PZ-27

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of water</td>
<td>( \rho_{\text{WATER}} ) (kg/m(^3))</td>
<td>998.2</td>
</tr>
<tr>
<td>Speed of sound, wat.</td>
<td>( c_{\text{WATER}} ) (m/s)</td>
<td>1478</td>
</tr>
<tr>
<td>Viscous loss factor</td>
<td>( \alpha_v ) (Np/m)</td>
<td>0.13</td>
</tr>
<tr>
<td>Density of steel</td>
<td>( \rho_{\text{STEEL}} ) (kg/m(^3))</td>
<td>8000</td>
</tr>
<tr>
<td>Density of PMMA</td>
<td>( \rho_{\text{PMMA}} ) (kg/m(^3))</td>
<td>1190</td>
</tr>
</tbody>
</table>

Table 2: Simulation data for PMMA, water and steel

The method described above is applicable only if the diffraction occurs mainly in one medium, i.e. if other parts of the propagation path are thin enough that diffraction in these can be considered negligible.
pulse is transmitted is modelled as transmission lines, with data calculated using methods from [6]. Simulation of the electronics is made using transistor models from the ASIC manufacturer. Estimated parasitic components are included in the simulation schematic. The temperature for the simulations is 20°C.

Parameter data for PZ-27 used in the simulation are shown in table 1 [10]. From [8] we get the equation for stiffened elastic constant

\[ C^D = C^E \left( 1 + \frac{(e^{33})^2}{e^8 C^E} \right). \]  

Van Deventer [6] developed expressions for the loss resistance in the model

\[ R = \frac{2\pi^2 f a^2 \rho}{Q}, \]  

the characteristic impedance

\[ Z_0 = \rho \pi a^2 c \]  

and the transmitting constant

\[ h = \frac{e^{33}}{e^5}. \]  

Further, the speed of sound is [8]:

\[ c = \sqrt{\frac{C^D}{\rho}}. \]  

Parameter data used for PMMA [6], water [6] and steel [11] are shown in table 2. From [6] we also get formulae for the calculation of backing and reflector impedances,

\[ Z = \rho c \pi a^2 \]  

and resistance to model viscoelastic loss in the transmission line:

\[ R = 2 \rho c \pi a^2 \alpha_v. \]  

**Experimental setup**

The system investigated is a pulse echo system, where the transducer is immersed in water. An ultrasonic pulse is sent towards a steel reflector and is received with the sending device. The piezoceramic element used is a PZ-27 disc manufactured by Ferroperm, Denmark. Two discs with diameters 6 mm and 12 mm have been used. The thickness is 0.5 mm, giving a fundamental frequency of approximately 4 MHz. The piezoceramic disc is mounted with cyanoacrylate glue on a carrier made of plexiglas (PMMA) to form a transducer unit. The plexiglas is used as backing and is shaped to reduce any echo returned to the rear side of the disc. The front side of the disc together with electrical connections is covered with one layer of PC-52 protective lacquer to allow immersion in water. The thin layer of lacquer does not significantly affect the measurements, and can thus be ignored in the simulations [12]. The transducer unit is mounted to a co-ordinate table, allowing both translation and rotational motion. The transducer was adjusted to be parallel to the steel reflector by observing when the returned echo amplitude was maximised. The electronics used is a custom-made ASIC driver chip [13]. The chip is a push-pull driver design, generating a square wave excitation pulse. The pulse width is set to half the crystal oscillating

![Figure 3: Measured echo amplitudes.](image-url)
period time to give maximum returned echo amplitude [14]. The chip is connected to the piezoelectric crystal with a 0.2m long coaxial cable. Measurements of the received echo signal are made on the chip side of the cable using a Tektronix TDS720 sampling oscilloscope.

III. MEASUREMENTS

Measurements were performed for two different transducers of diameters 6 mm and 12 mm. The thickness of the discs is 0.5 mm, giving a resonance frequency of about 4 MHz. Results from the measurements are shown in figure 3. The far field limit where the sound pulse starts a more rapid divergence is clearly seen for the smaller transducer at a reflector distance of 30 mm. The far field limit for the larger transducer is about 120 mm. Water can be considered to be a good carrier of ultrasonic waves, with attenuation due to viscoelastic losses of about 1.1 dB/m [6]. This verifies that the measured attenuation profiles presented in figure 3 are indeed produced by diffraction effects.

IV. SIMULATIONS

Initial simulations were made with the diameter of the piezoelectric disc set to 6 mm and 12 mm. Results from these simulations are shown in figure 4 and 5 respectively, together with measured results. From the outcome of these simulations, it was clear that the diffraction loss was underestimated as the whole curve was shifted towards too high values along the x-axis. In the simulation model, the parameter $S$, as defined in (8), is decisive for the behaviour of the curve. $S$ in turn is highly dependant on the transducer radius, $a$. If $a$ is decreased, the $S$ values will increase and the curves will be shifted towards smaller values along the x-axis. Thus, the diameter used in the simulation was decreased in order to find a better match to the measurements. From the initial results it was seen that the curve should be shifted 25-30% to the left. This corresponds to a decrease in diameter of 13-16%. It was found that decreasing the diameter about 13% gave a good match to the measured data. Simulated results with diameters set to 5.2 mm and 10.4 mm are plotted in figures 4 and 5 respectively.

Sensitivity analysis

When aiming for absolute amplitude correctness in simulations, the material parameters used need to be correct. The parameters affecting the outcome of the simulations were investigated. Values were increased with 10%, and the resulting change in received echo amplitude was recorded. For these simulations no attenuation due to diffraction was used. Also, a lossless model was used for water in order to isolate effects from the piezoceramic disc. The pulse width of the excitation pulse was adjusted to remain at half the crystal oscillating period time.
for all simulations. Results from the simulations are shown in table 3. Here is seen that variations in material parameters can have major effect on the result. As several of the parameters presented for PZ-27 have an uncertainty of 10%, this should be considered when interpreting results from simulations.

V. CONCLUSIONS

This paper has presented a method for the inclusion of diffraction loss in Pspice simulations of ultrasonic systems. Conductive loss in the transmission line modelling the propagation media of the ultrasonic pulse is used to model the diffraction loss. The method can be used either for systems employing separate transmitter/receivers of equal size, or in a pulse echo system. The method is limited to systems where the diffraction loss occurs mainly in one part of the transmission path.

For a pulse echo system in water, good agreement is shown between simulated and measured results. Simulated echo amplitudes are within 10% of measured data in both near and far field. However, an adjustment is needed in the effective radius of the transducer, such that the value used in simulations should be 13% lower than actual used radius.

VI. REFERENCES


