Assignment 1

Ex1. Given a mathematical notation, write corresponding terms in λ-calculus:

(a) f(x), *where* f(x) = x + 3

(b) f(y), *where* f(y) = x + y

(c) f(a,b), *where* f(a,b) = a + b – x

(d) f(x,y), *where* f(x,y) = x + (y – x) + g(y) *and* g(x) = x + 4

(e) f(x,y), *where* f(x,y) = x + (y – x) + g(y) *and* g(x) = x + y

(f) f(x,g), *where* f(x,g) = g(g(x))

(g) f(12), *where* f(x) = x + 3

(h) f(5), *where* f(x) = y – x

(i) f(1,3), *where* f(x,y) = x – y

(j) f(2,g), *where* f(x,y) = y(x)

(k) f(7,g), *where* f(x,y) = y(y(x)+x) *and* g(x) = x – 3

(l) f(a,2), *where* f(x,y) = x(y) + x(y – 1)

Ex 2. Evaluate the following terms in λ-calculus:

(a) (λx. x + 4) 5

(b) (λx. λy. x – y) 3 1

(c) (λx. λy. x + (λz. z + z) y) 10 7

(d) (λa. λf. 10 + f a + f (f a)) 5 (λx. x + x)

(e) (λz. 5 + (λx. λy. y x + z) 7 (λz. z+2)) 1

(f) (λx. x + 7 – (3 + x)) 25 + (λy. y 5) (λx. x – 3)

Ex3. Extend untyped λ-calculus defined in the slides by adding boolean values (*true* and *false*), operations on booleans (“and” and “or”), and *if*-expressions (*if t then* t_1 *else* t_2). Define the syntax and operational semantics assuming *lazy evaluation* of *if*-expressions, i.e. that the term t_1 (or t_2) is only evaluated after the condition *t* has been evaluated to a value.