Overview

• Preliminaries
• Clipping
  – Line clipping
• Hidden Surface removal

Overview

• At end of the geometric pipeline, vertices have been assembled into primitives
• Must clip out primitives that are outside the view frustum
  – Algorithms based on representing primitives by lists of vertices
• Must find which pixels can be affected by each primitive
  – Fragment generation
  – Rasterization or scan conversion

Required Tasks

• Clipping
  – This lecture
• Rasterization or scan conversion
  – Next lecture
• Transformations
• Some tasks deferred until fragment processing
  – Hidden surface removal
  – Antialiasing
Rasterization Meta Algorithms

- Two approaches
- For every pixel, determine which object that projects on the pixel is closest to the viewer and compute the shade of this pixel
  - Ray tracing paradigm
  - Global effects
- For every object, determine which pixels it covers and shade these pixels
  - Pipeline approach
  - Must keep track of depths
  - No Global effects

Clipping

- 2D against clipping window
- 3D against clipping volume
- Easy for line segments polygons
- Hard for curves and text
  - Convert to lines and polygons first

Clipping 2D Line Segments

- Brute force approach: compute intersections with all sides of clipping window
  - Inefficient: one division per intersection

Cohen-Sutherland Algorithm

- Idea: eliminate as many cases as possible without computing intersections
- Start with four lines that determine the sides of the clipping window

\[
\begin{align*}
  x &= x_{\text{min}} \\
  x &= x_{\text{max}} \\
  y &= y_{\text{min}} \\
  y &= y_{\text{max}}
\end{align*}
\]
The Cases

- Case 1: both endpoints of line segment inside all four lines
  - Draw (accept) line segment as is
    \[
    \begin{array}{c|c}
    x = x_{\text{min}} & x = x_{\text{max}} \\
    \hline
    y = y_{\text{min}} & y = y_{\text{max}}
    \end{array}
    \]

- Case 2: both endpoints outside all lines and on same side of a line
  - Discard (reject) the line segment

- Case 3: One endpoint inside, one outside
  - Must do at least one intersection

- Case 4: Both outside
  - May have part inside
  - Must do at least one intersection

Defining Outcodes

- For each endpoint, define an outcode
  \[
  b_0b_1b_2b_3
  \]
  \[
  \begin{array}{l}
  b_0 = 1 \text{ if } y > y_{\text{max}}, 0 \text{ otherwise} \\
  b_1 = 1 \text{ if } y < y_{\text{min}}, 0 \text{ otherwise} \\
  b_2 = 1 \text{ if } x > x_{\text{max}}, 0 \text{ otherwise} \\
  b_3 = 1 \text{ if } x < x_{\text{min}}, 0 \text{ otherwise}
  \end{array}
  \]

- Outcodes divide space into 9 regions
- Computation of outcode requires at most 4 subtractions

Using Outcodes

- Consider the 5 cases below
- AB: outcode(A) = outcode(B) = 0
  - Accept line segment
Using Outcodes

• CD: outcode (C) = 0, outcode(D) ≠ 0
  – Compute intersection
  – Location of 1 in outcode(D) determines which edge to intersect with
  – Note if there were a segment from A to a point in a region with 2 ones in outcode, we might have to do two interesections

Using Outcodes

• EF: outcode(E) logically ANDed with outcode(F) (bitwise) ≠ 0
  – Both outcodes have a 1 bit in the same place
  – Line segment is outside of corresponding side of clipping window
  – reject

Using Outcodes

• GH and IJ: same outcodes, neither zero but logical AND yields zero

• Shorten line segment by intersecting with one of sides of window

• Compute outcode of intersection (new endpoint of shortened line segment)

• Reexecute algorithm

Efficiency

• In many applications, the clipping window is small relative to the size of the entire data base
  – Most line segments are outside one or more side of the window and can be eliminated based on their outcodes

• Inefficiency when code has to be reexecuted for line segments that must be shortened in more than one step
Cohen Sutherland in 3D

- Use 6-bit outcodes
- When needed, clip line segment against planes

Liang-Barsky Clipping

- Consider the parametric form of a line segment
  \[ p(\alpha) = (1-\alpha)p_1 + \alpha p_2 \quad 1 \geq \alpha \geq 0 \]

- We can distinguish between the cases by looking at the ordering of the values of \( \alpha \) where the line determined by the line segment crosses the lines that determine the window

Liang-Barsky Clipping

- In (a): \( \alpha_4 > \alpha_3 > \alpha_2 > \alpha_1 \)
  - Intersect right, top, left, bottom: shorten
- In (b): \( \alpha_4 > \alpha_2 > \alpha_3 > \alpha_1 \)
  - Intersect right, left, top, bottom: reject

Advantages

- Can accept/reject as easily as with Cohen-Sutherland
- Using values of \( \alpha \), we do not have to use algorithm recursively as with C-S
- Extends to 3D
Clipping and Normalization

- General clipping in 3D requires intersection of line segments against arbitrary plane
- Example: oblique view

![Clipping Diagram]

Normalized Form

![Normalized Form Diagram]

Normalization is part of viewing (pre clipping) but after normalization, we clip against sides of right parallelepiped

Typical intersection calculation now requires only a floating point subtraction, e.g. is \( x > x_{\text{max}} \) ?

Plane-Line Intersections

\[
a = \frac{n \cdot (p_o - p_1)}{n \cdot (p_2 - p_1)}
\]

Polygon Clipping

- Not as simple as line segment clipping
  - Clipping a line segment yields at most one line segment
  - Clipping a polygon can yield multiple polygons

- However, clipping a convex polygon can yield at most one other polygon
Tessellation and Convexity

- One strategy is to replace nonconvex (concave) polygons with a set of triangular polygons (a tessellation)
- Also makes fill easier
- Tessellation code in GLU library

Clipping as a Black Box

- Can consider line segment clipping as a process that takes in two vertices and produces either no vertices or the vertices of a clipped line segment

Pipeline Clipping of Line Segments

- Clipping against each side of window is independent of other sides
  - Can use four independent clippers in a pipeline

Pipeline Clipping of Polygons

- Three dimensions: add front and back clippers
- Strategy used in SGI Geometry Engine
- Small increase in latency
### Bounding Boxes

- Rather than doing clipping on a complex polygon, we can use an *axis-aligned bounding box* or *extent*
  - Smallest rectangle aligned with axes that encloses the polygon
  - Simple to compute: max and min of x and y

### Clipping and Visibility

- Clipping has much in common with hidden-surface removal
- In both cases, we are trying to remove objects that are not visible to the camera
- Often we can use visibility or occlusion testing early in the process to eliminate as many polygons as possible before going through the entire pipeline

### Hidden Surface Removal

- Object-space approach: use pairwise testing between polygons (objects)
- Worst case complexity $O(n^2)$ for $n$ polygons
**Painter’s Algorithm**

- Render polygons a back to front order so that polygons behind others are simply painted over

  ![Diagram of Painter's Algorithm]

  B behind A as seen by viewer  
  Fill B then A

**Depth Sort**

- Requires ordering of polygons first
  - $O(n \log n)$ calculation for ordering
  - Not every polygon is either in front or behind all other polygons

- Order polygons and deal with easy cases first, harder later

  ![Diagram of Depth Sort]

  Polygons sorted by distance from COP

**Easy Cases**

- A lies behind all other polygons
  - Can render

- Polygons overlap in z but not in either x or y
  - Can render independently

**Hard Cases**

- Overlap in all directions but can one is fully on one side of the other

- Cyclic overlap

- Penetration
Back-Face Removal (Culling)

• face is visible iff $90 \geq \theta \geq -90$
  equivalently $\cos \theta \geq 0$
  or $v \cdot n \geq 0$

• plane of face has form $ax + by + cz + d = 0$
  but after normalization $n = (0 \ 0 \ 1 \ 0)^T$

• need only test the sign of $c$

• In OpenGL we can simply enable culling
  but may not work correctly if we have nonconvex objects

Image Space Approach

• Look at each projector ($nm$ for an $n \times m$
  frame buffer) and find closest of $k$
  polygons

• Complexity $O(nmk)$

• Ray tracing

• $z$-buffer

z-Buffer Algorithm

• Use a buffer called the $z$ or depth buffer to store
  the depth of the closest object at each pixel
  found so far

• As we render each polygon, compare the depth
  of each pixel to depth in $z$ buffer

• If less, place shade of pixel in color buffer and
  update $z$ buffer

Efficiency

• If we work scan line by scan line as we
  move across a scan line, the depth
  changes satisfy $a\Delta x + b\Delta y + c\Delta z = 0$

  Along scan line
  $\Delta y = 0$
  $\Delta z = -\frac{a}{c} \Delta x$

  In screen space $\Delta x = 1$
Visibility Testing

- In many realtime applications, such as games, we want to eliminate as many objects as possible within the application
  - Reduce burden on pipeline
  - Reduce traffic on bus
- Partition space with Binary Spatial Partition (BSP) Tree
- Octrees
- Portals

Simple Example

Consider 6 parallel polygons

The plane of A separates B and C from D, E and F

BSP Tree

- Can continue recursively
  - Plane of C separates B from A
  - Plane of D separates E and F
- Can put this information in a BSP tree
  - Use for visibility and occlusion testing

Next time

- Scan conversion and Rasterization